# psyc3010 lecture 3 extra materials

## calculations in 2-way ANOVA: follow-up tests

This material is provided to give you a deeper understanding of ANOVA, ***if*** you find it useful to look at the formulae and calculations. It is optional material.

If you have questions, you are very welcome to ask!

# back to the example from Lecture 2: finding the mystery

***main effect of Factor A (alcohol consumption):***

$H_0$: $\mu_1 = \mu_2 = \mu_3$

*reject* $H_0$ if $MS_A$ / $MS_{error}$ results in a significant obtained F value

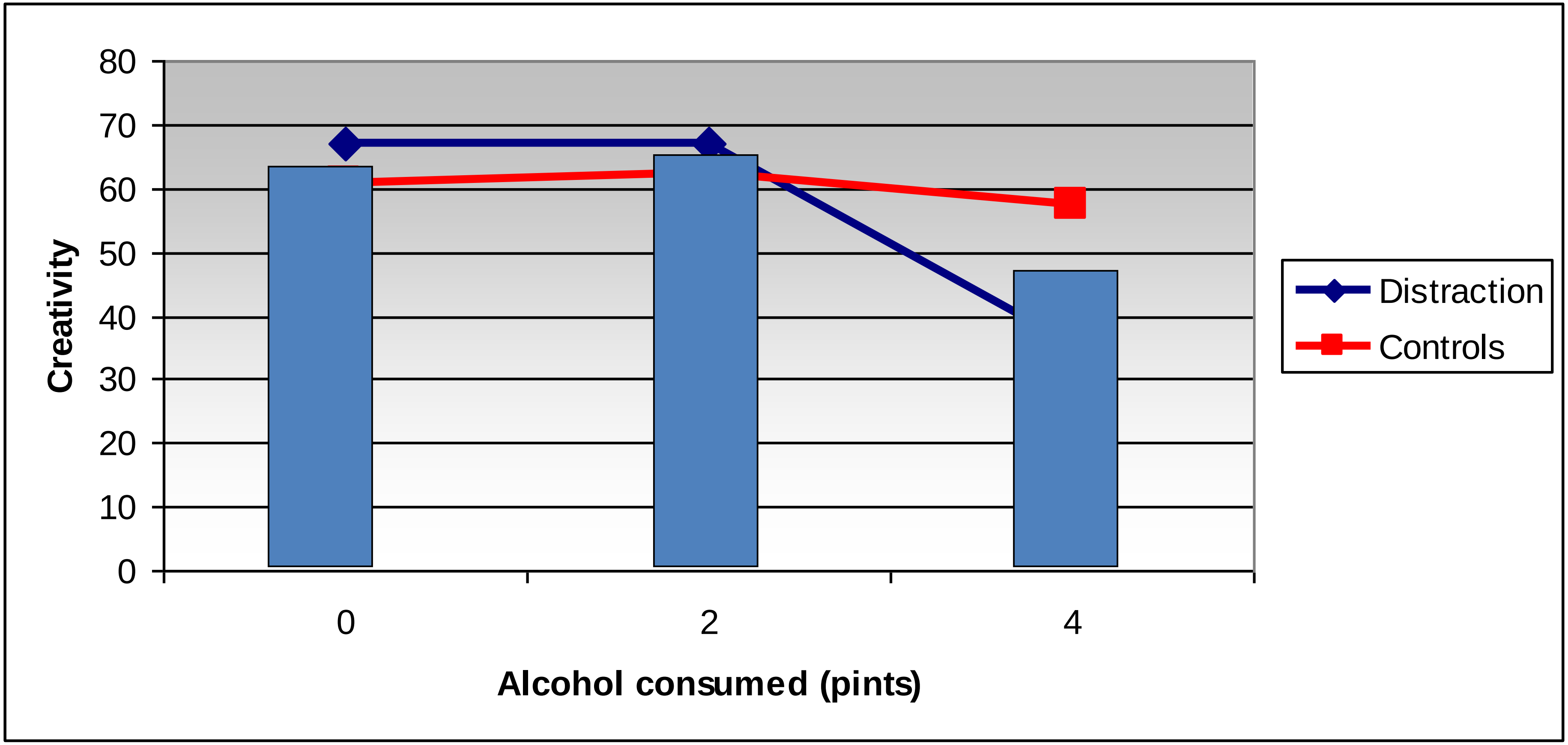
**$F$ (2, 42) = 20.06, $p$ < .001**

→indicates that the 3 levels of Factor A differ (collapsed across factor B)

→indicates that the marginal means of Factor A differ

| Participant Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (B) (means) |
|---|---|---|---|---|
| | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| **Distraction** | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | **535** | **535** | **285** | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |
| | 65 | 70 | 55 | |
| | 70 | 65 | 65 | |
| | 60 | 60 | 70 | |
| **Controls** | 60 | 70 | 55 | |
| | 60 | 65 | 55 | |
| | 55 | 60 | 60 | |
| | 60 | 60 | 50 | |
| | 55 | 50 | 50 | |
| Cell Totals | **485** | **500** | **460** | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (*A*) | **1020** | **1035** | **745** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# main effect of alcohol consumed



## ***interaction of A x B***

$H_0: \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$

*reject* $H_0$ if $MS_{AB} / MS_{error}$ results in a significant obtained F

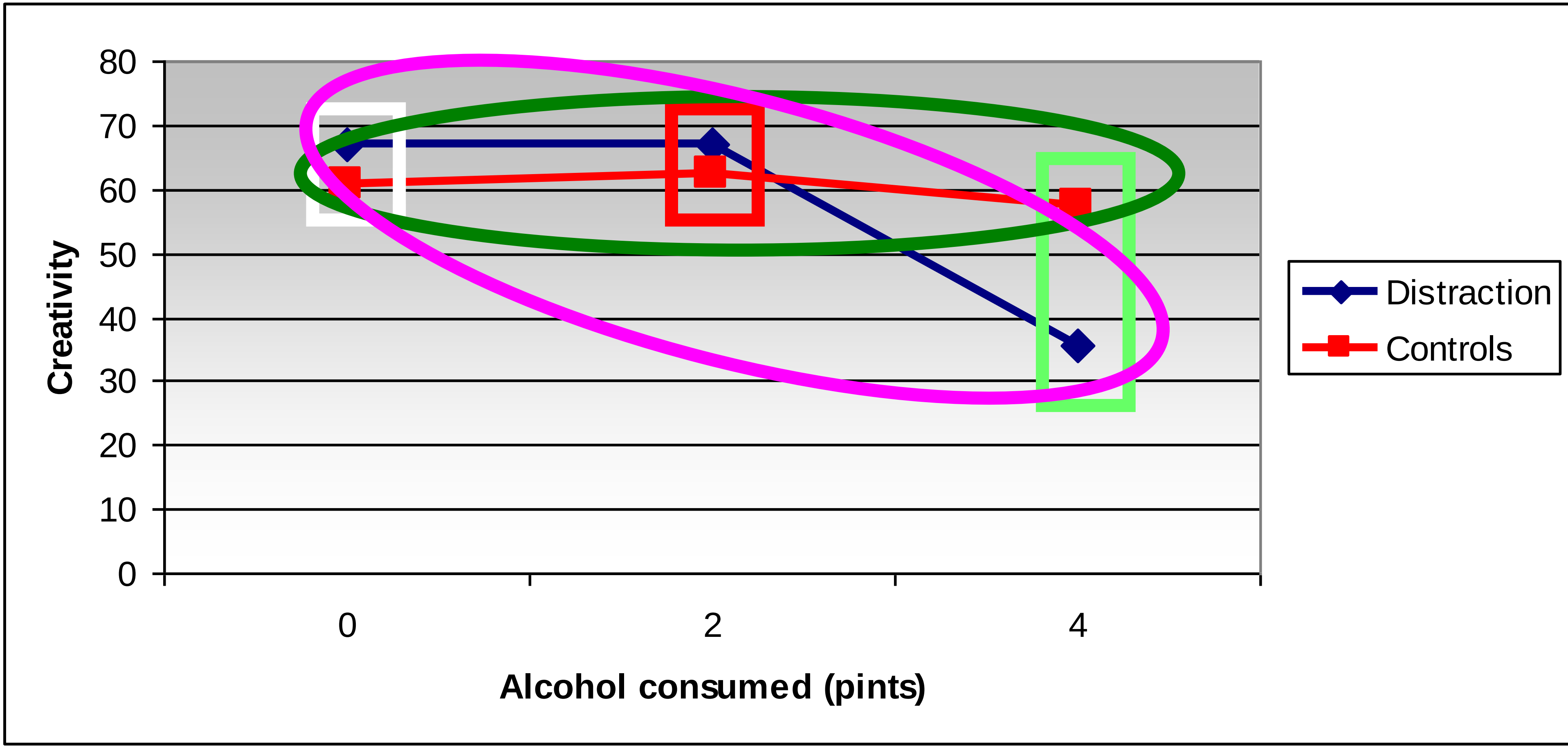
$F(2,42) = 11.91$, $p < .001$

→ indicates that the effect of factor B is not the same at all levels of factor A (or vice versa)

→ difference in cell means for levels of one factor changes depending on level of other factor

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (B) (means) |
|---|---|---|---|---|
| | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| **Distraction** | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | **535** | **535** | **285** | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |
| **Controls** | | | | |
| Cell Totals | | | | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (*A*) | ***1020*** | ***1035*** | ***745*** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# interaction



# following up main effects

## protected *t*-tests & linear contrasts

# following up main effects:
# (differences among *marginal means*)

- the ***"protected t-test"*** is used to conduct pairwise comparisons (i.e., compare 2 means), but ***only if the main effect is significant***
  - e.g., to compare effect of 4 pints to 2 pints
  - *n* must be based on the number of observations in each level we're comparing [n X number of levels of other IV]

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{2MS_{error}}{n \times d}}}$$

This formula would be what you could use to follow up the **MAIN EFFECT OF ALCOHOL**

$$df_{error} = N - ab$$

# **following up main effects:**
# (differences among *marginal means*)

- the ***"protected t-test"*** is used to conduct pairwise comparisons (i.e., compare 2 means), but ***only if the main effect is significant***
  - e.g., to compare effect of 4 pints to 2 pints
  - *n* must be based on the number of observations in each level we're comparing [n X number of levels of the other IV]

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{2MS_{error}}{n \times d}}}$$

The **d** here represents "number of levels of the **d**istraction variable"

→ but you could change the letter

$$df_{error} = N - ab$$

to follow up our main effect of A (alcohol consumption)

*"are creativity ratings lower after 4 pints than after 0 pints?"*

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{2MS_{error}}{n \times d}}} \rightarrow t = \frac{46.56 - 63.75}{\sqrt{\frac{2 \times 83.02}{8 \times 2}}}$$

$$df_{error} = N - ab$$

$$|t_{obt}(42) = -5.34| > |t_{crit}(42) = \pm 2.021|$$

***"Yes, there is a significant difference"***

| **Distraction** | **Alcohol Consumption (pints) 0** | **2** | **4** | Marginal Totals (B) (means) |
|---|---|---|---|---|
| | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| **Distraction** | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | **535** | **535** | **285** | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |
| | 65 | 70 | 55 | |
| | 70 | 65 | 65 | |
| | 60 | 60 | 70 | |
| **Controls** | 60 | 70 | 55 | |
| | 60 | 65 | 55 | |
| | 55 | 60 | 60 | |
| | 60 | 60 | 50 | |
| | 55 | 50 | 50 | |
| Cell Totals | **485** | **500** | **460** | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (*A*) | ***1020*** | ***1035*** | ***745*** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# following up main effects:
## (differences among *marginal means*)

- as an alternative, could use **Linear Contrasts** to determine if one group ***or set of groups*** is different from another group ***or set of groups***
- a set of weights, $a_j$, is used to define the contrast

  e.g., $\bar{X}_1$ & $\bar{X}_2$ vs. $\bar{X}_3$ → **1 1 -2**
- (the protected t-test is a special case of this technique)

$$t = \frac{L}{\sqrt{\dfrac{\sum a_j^2 MS_{error}}{n \times no.levels.of.other.IV}}}$$

$$L = \sum a_j \bar{X}_j$$

$$df_{error} = N - ab$$

# following up interactions part 1

## simple effects

# how we test the simple effects

***say we're testing the simple effects of Factor A...***

1. **calculate $SS_{treatment}$ for Factor A at first level of Factor B**
   - **$SS_{treatment}$** for simple effects = variability of cell means: Factor A in one level of Factor B

2. **calculate $MS_{treatment}$, using the degrees of freedom (DF) for the omnibus main effect of Factor A**
   - i.e., from original ANOVA summary table

3. **use $MS_{error}$ from omnibus tests**
   - i.e., from the original ANOVA summary table

4. **calculate F ratio: $F = MS_{treatment} / MS_{error}$**

5. **repeat for each remaining level of Factor B**

# simple effects of distraction

## "what is the effect of distraction at each level of consumption?"

is there an effect of distraction for participants who have consumed….

0 pints?

2 pints?

4 pints?

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (B) (means) |
|---|---|---|---|---|
| | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| **Distraction** | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | [illegible] | [illegible] | [illegible] | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |
| **Controls** | | | | |
| Cell Totals | | | | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (*A*) | ***1020*** | ***1035*** | ***745*** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# simple effects of distraction

## effect after 0 pints

$$SS_{Distraction.at.Consumption1} = \frac{\sum T_{D.at.C1}^2}{n} - \frac{T_{C1}^2}{nd}$$

$$= \frac{535^2 + 485^2}{8} - \frac{1020^2}{16}$$

$= 156.25$

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (D) |
|---|---|---|---|---|
| **Distraction** | 50 | | | |
| | 55 | | | |
| | 80 | | | |
| | 65 | | | |
| | 70 | | | |
| | 75 | | | |
| | 75 | | | |
| | 65 | | | |
| Cell Totals | **535** | | | |
| Cell Means | ***66.88*** | | | |
| **Controls** | 65 | | | |
| | 70 | | | |
| | 60 | | | |
| | 60 | | | |
| | 60 | | | |
| | 55 | | | |
| | 60 | | | |
| | 55 | | | |
| Cell Totals | **485** | | | |
| Cell Means | ***60.63*** | | | |
| Marginal Totals (C) | ***1020*** | | | |
| Means | ***63.75*** | | | |

**<u>NOTE</u>:**
**to help remember which factor we are talking about, we can use labels other than A and B**

**– e.g.:**
**D = distraction**
**C = consumption**

# simple effects of distraction

## effect after 2 pints

$$SS_{Distraction.at.Consumption2} = \frac{\sum T_{D.at.C_2}^{\;2}}{n} - \frac{T_{C_2}^2}{nd}$$

$$= \frac{535^2 + 500^2}{8} - \frac{1035^2}{16}$$

$$= 76.56$$

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (D) |
|---|---|---|---|---|
| **Distraction** | | 45 | | |
| | | 60 | | |
| | | 85 | | |
| | | 65 | | |
| | | 70 | | |
| | | 70 | | |
| | | 80 | | |
| | | 60 | | |
| Cell Totals | | **535** | | |
| Cell Means | | ***66.88*** | | |
| **Controls** | | 70 | | |
| | | 65 | | |
| | | 60 | | |
| | | 70 | | |
| | | 65 | | |
| | | 60 | | |
| | | 60 | | |
| | | 50 | | |
| Cell Totals | | **500** | | |
| Cell Means | | ***62.50*** | | |
| Marginal Totals (C) | | ***1035*** | | |
| Means | | ***64.69*** | | |

# simple effects of distraction

## effect after 4 pints

$$SS_{Distraction.at.Consumption3} = \frac{\sum T_{D.at.C_3}^{2}}{n} - \frac{T_{C_3}^{2}}{nd}$$

$$= \frac{285^2 + 460^2}{8} - \frac{745^2}{16}$$

$$= 1914.06$$

| **Distraction** | **Alcohol Consumption (pints)** | | | Marginal |
|---|---|---|---|---|
| | **0** | **2** | **4** | Totals (D) |
| | | | 30 | |
| | | | 30 | |
| | | | 30 | |
| | | | 55 | |
| **Distraction** | | | 35 | |
| | | | 20 | |
| | | | 45 | |
| | | | 40 | |
| Cell Totals | | | **285** | |
| Cell Means | | | ***35.63*** | |
| | | | 55 | |
| | | | 65 | |
| | | | 70 | |
| **Controls** | | | 55 | |
| | | | 55 | |
| | | | 60 | |
| | | | 50 | |
| | | | 50 | |
| Cell Totals | | | **460** | |
| Cell Means | | | ***57.50*** | |
| Marginal Totals (C) | | | ***745*** | |
| Means | [illegible] | [illegible] | ***46.56*** | [illegible] |

# summary table for simple effects of distraction

| **Source** | ***SS*** | ***df*** | ***MS*** | ***F*** | ***p*** |
|---|---|---|---|---|---|
| **D at C1** | 156.25 | 1 | 156.25 | 1.88 | 0.177 |
| **D at C2** | 76.56 | 1 | 76.56 | 0.92 | 0.342 |
| **D at C3** | 1914.06 | 1 | 1914.06 | 23.05 | 0.000 |
| **Error** | 3487.5 | 42 | 83.04 | | |

critical *F* at alpha=.05 (1,42) = 4.08

if obtained F exceeds critical F ***reject the null hypothesis*** these are your calculated **SS** values

**degrees of freedom** for a simple effect are just the *df* for the associated main effect

$df = df_{distraction}$ (2 - 1) = 1

| Source | *SS* | *df* | *MS* | *F* | *p* |
|---|---|---|---|---|---|
| **D at C1** | **156.25** | **1** | **156.25** | **1.88** | 0.177 |
| **D at C2** | **76.56** | **1** | **76.56** | **0.92** | 0.342 |
| **D at C3** | **1914.06** | **1** | **1914.06** | **23.05** | 0.000 |
| **Error** | **3487.5** | **42** | **83.04** | | |

critical F at alpha=.05 (1,42) = 4.08

if obtained F exceeds critical F ***reject the null hypothesis***

**mean squares** and ***F* values** calculated as SS / df and $MS_{effect}$ / $MS_{error}$

$SS_{error}$ term (and *df*) is taken from the main ANOVA (calculated last week)

# simple effects of consumption

"what is the effect of consumption at each level of distraction?"

is there an effect of consumption for….

distracted?

controls?

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (D) (means) |
|---|---|---|---|---|
| | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| **Distraction** | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | **535** | **535** | **285** | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |
| | 65 | 70 | 55 | |
| | 70 | 65 | 65 | |
| | 60 | 60 | 70 | |
| **Controls** | 60 | 70 | 55 | |
| | 60 | 65 | 55 | |
| | 55 | 60 | 60 | |
| | 60 | 60 | 50 | |
| | 55 | 50 | 50 | |
| Cell Totals | **485** | **500** | **460** | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (C) | ***1020*** | ***1035*** | ***745*** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# simple effects of consumption

## effect when distracted

$$SS_{Consumption.at.Distraction_1} = \frac{\sum T_{C.at.D_1}^2}{n} - \frac{T_{D_1}^2}{nc}$$

$$= \frac{535^2 + 535^2 + 285^2}{8} - \frac{1355^2}{24}$$

= 5208.33

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (D) (means) |
|---|---|---|---|---|
| Distraction | 50 | 45 | 30 | |
| | 55 | 60 | 30 | |
| | 80 | 85 | 30 | |
| | 65 | 65 | 55 | |
| | 70 | 70 | 35 | |
| | 75 | 70 | 20 | |
| | 75 | 80 | 45 | |
| | 65 | 60 | 40 | |
| Cell Totals | **535** | **535** | **285** | ***1355*** |
| Cell Means | ***66.88*** | ***66.88*** | ***35.63*** | ***56.46*** |

# simple effects of consumption

effect in control group

$$SS_{Consumption.at.distraction_2} = \frac{\sum T_{C.at.D_2}{}^2}{n} - \frac{T_{D_2}^2}{nc}$$

$$= \frac{485^2 + 500^2 + 460^2}{8} - \frac{1445^2}{24}$$

**= 102.08**

| Distraction | Alcohol Consumption (pints) 0 | 2 | 4 | Marginal Totals (D) (means) |
|---|---|---|---|---|
| | 65 | 70 | 55 | |
| | 70 | 65 | 65 | |
| | 60 | 60 | 70 | |
| **Controls** | 60 | 70 | 55 | |
| | 60 | 65 | 55 | |
| | 55 | 60 | 60 | |
| | 60 | 60 | 50 | |
| | 55 | 50 | 50 | |
| Cell Totals | **485** | **500** | **460** | ***1445*** |
| Cell Means | ***60.63*** | ***62.50*** | ***57.50*** | ***60.21*** |
| Marginal Totals (C) | ***1020*** | ***1035*** | ***745*** | ***2800*** |
| Means | ***63.75*** | ***64.69*** | ***46.56*** | ***58.33*** |

# summary table for simple effects of consumption

| **Source** | ***SS*** | ***df*** | ***MS*** | ***F*** | ***p*** |
|---|---|---|---|---|---|
| **C at D1** | 5208.33 | 2 | 2604.17 | 31.36 | 0.000 |
| **C at D2** | 102.08 | 2 | 51.04 | 0.61 | 0.546 |
| **Error** | 3487.5 | 42 | 83.04 | | |

critical F at alpha=.05 (242) = 3.23

if obtained F exceeds critical F ***reject the null hypothesis*** degrees of freedom for a simple effect are just the *df* for the associated main effect

$df = df_{consumption} (3 - 1) = 2$

these are your calculated SS values

| Source | *SS* | *df* | *MS* | *F* | *p* |
|---|---|---|---|---|---|
| **C at D1** | **5208.33** | **2** | **2604.17** | **31.36** | 0.000 |
| **C at D2** | **102.08** | **2** | **51.04** | **0.61** | 0.546 |
| **Error** | **3487.5** | **42** | **83.04** | | |

critical F at alpha=.05 (242) = 3.23

if obtained F exceeds critical F ***reject the null hypothesis***

mean squares and *F* values calculated as per last week

$SS_{error}$ term (and *df*) is taken from the main ANOVA (calculated last week)

# following up interactions part 2

## simple comparisons

# following up simple effects:
## linear contrasts and simple comparisons

- **consider the significant simple effect of consumption for *distracted participants*:**
  – indicates that, *for distracted*, there is a difference among the means over the 3 levels of consumption
  (0 pints, 2 pints, 4 pints)

- **follow-up using simple comparisons (linear contrasts)**
  – the procedure is *identical* to that used for following up main effects, except comparisons are between **cell means**, not marginal means

→ *NOTE:* only ***significant simple effects*** should be followed up

# simple comparisons for consumption (distracted)

| | Consumption | | |
|---|---|---|---|
| | 0 pints | 2 pints | 4 pints |
| **Distracted** | 66.88 | 66.88 | 35.63 |
| Contrast 1 | 2 | -1 | -1 |
| Contrast 2 | 0 | 1 | -1 |

these are the **cell means for distracted participants** from our data table earlier

a set of weights ($a_j$) is used to define the contrasts:

contrast 1 compares 0 vs 2 & 4

contrast 2 compares 2 vs 4

| | Consumption | | |
|---|---|---|---|
| | 0 pints | 2 pints | 4 pints |
| **Distracted** | **66.88** | **66.88** | **35.63** |
| Contrast 1 | **2** | **-1** | **-1** |
| Contrast 2 | **0** | **1** | **-1** |

contrasts are orthogonal:

$\sum a_j = 0$ *[sum of contrasts within a]*

$\sum a_j b_j = 0$ *[sum of products of each j set of contrasts]*

number of contrasts = df for effect

# **calculations** for contrast 1

$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n}}}$$

$$L = \sum a_j \overline{X}_j$$

$$df_{error} = N - ab$$

| | Consumption | | |
|---|---|---|---|
| | 0 pints | 2 pints | 4 pints |
| **Distracted** | **66.88** | **66.88** | **35.63** |
| Contrast 1 | **2** | **-1** | **-1** |
| Contrast 2 | 0 | 1 | -1 |

***L*** = 2(66.88) – 1(66.88) – 1(35.63) = ***31.25***

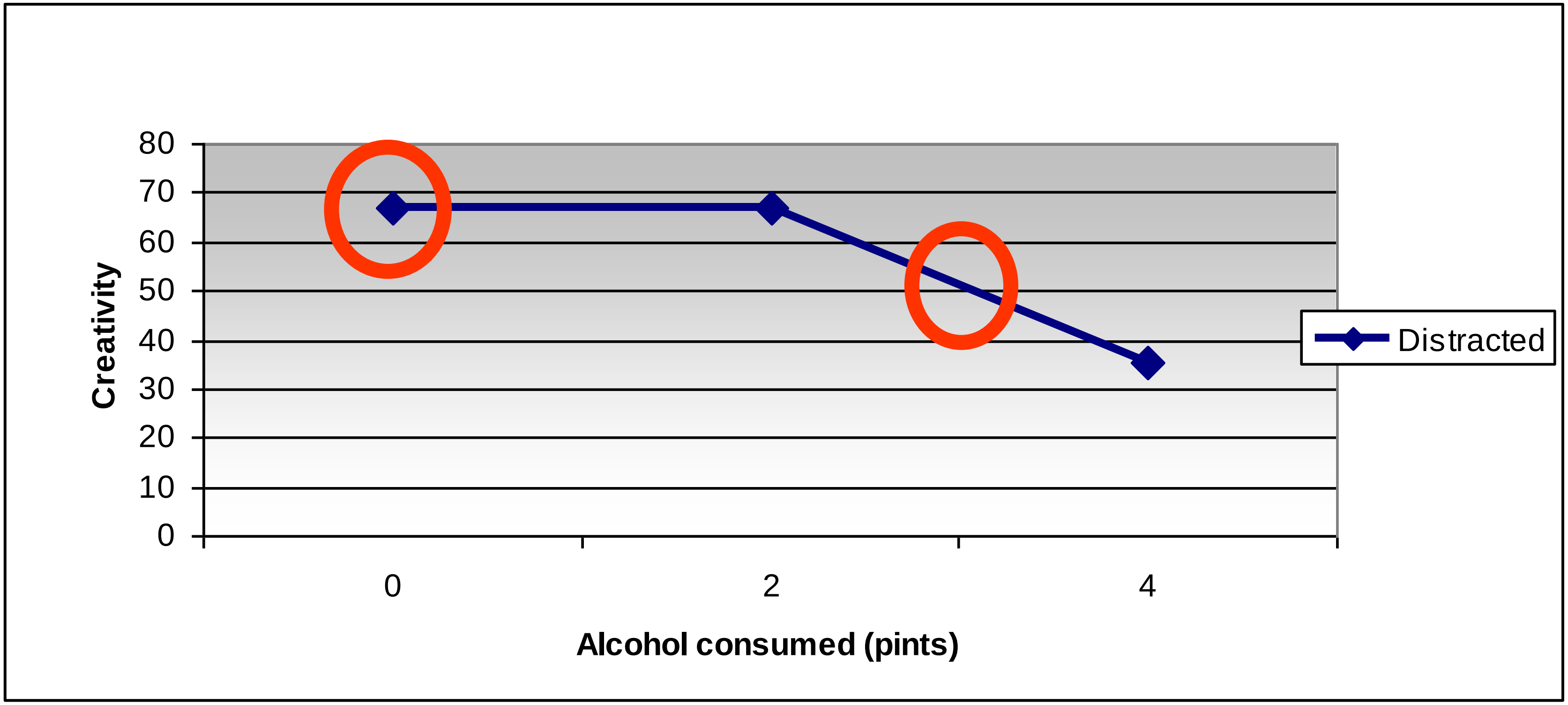
$$t = \frac{31.25}{\sqrt{\frac{(2^2 + (-1)^2 + (-1)^2)83.04}{8}}} = 3.96$$

$t_{\alpha=.05}$ (42) = 2.02 (unadjusted)

$t_{\alpha=.05}$ (42) = 2.33 (adjusted)

(Bonferroni adjustment for 2 comparisons)

# contrast 1 – what does it do?




***contrast 1*** **compares *(for distracted participants only)* the mean creativity rating for participants who have had *0 pints* with the mean attractiveness rating for participants who have had *2 or 4 pints:*** $t\,(42) = 3.96$, $p < .05$ → significant

# calculations for contrast 2

$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n}}}$$

$$L = \sum a_j \bar{X}_j$$

$$df_{error} = N - ab$$

| | Consumption | | |
|---|---|---|---|
| | 0 pints | 2 pints | 4 pints |
| **Distracted** | **66.88** | **66.88** | **35.63** |
| Contrast 1 | 2 | -1 | -1 |
| Contrast 2 | **0** | **1** | **-1** |

$$\boldsymbol{L} = 0(66.88) + 1(66.88) - 1(35.63) = \boldsymbol{31.25}$$

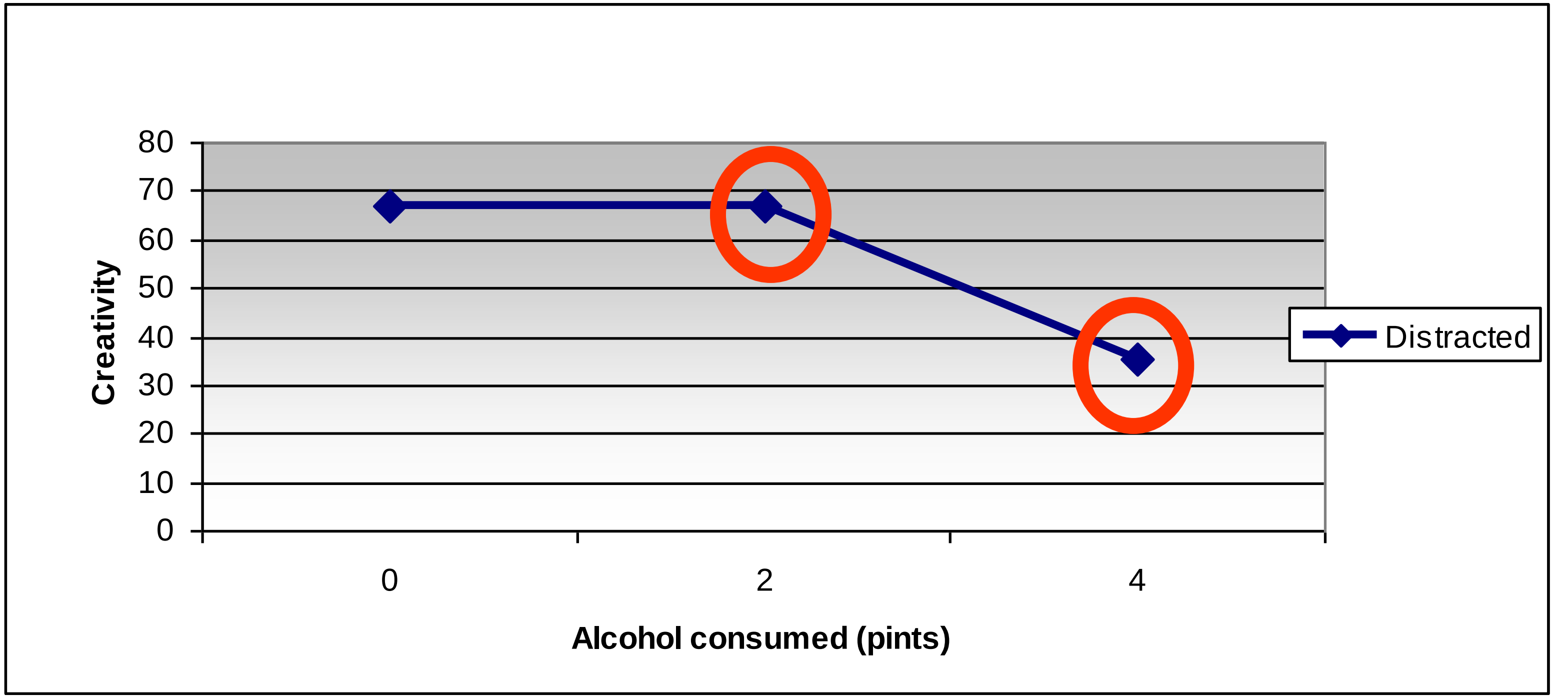
$$t = \frac{31.25}{\sqrt{\frac{(0^2 + 1^2 + (-1)^2)83.04}{8}}} = 6.86$$

$t_{\alpha=.05}(42) = 2.02$ (unadjusted)

$t_{\alpha=.05}(42) = 2.33$ (adjusted)

(Bonferroni adjustment for 2 comparisons)

# contrast 2 – what does it do?




***contrast 2*** **compares *(for distracted participants only)* the mean creativity rating for participants who have had *2 pints* with the mean attractiveness rating for participants who have had *4 pints:* $t$ (42) = 6.86, $p$ < .05** → significant

# Another Example of Follow-Up Tests Using Linear Contrasts (Based on an Adapted Version of the Workbook Data)

# hypothesised effect of drug dosage

- this example uses the data in your tutorial workbooks
- let's say that Hypothesis 2 was different from the one presented in your tute workbooks

Instead the NEW Hypothesis 2 we'll be working with for this is:

Overall, rats will perform better when they receive the drug than when they do not receive the drug. However, a small dose will tend to lead to the best performance (compared to moderate and large doses).

# from hypotheses to analyses: developing the analysis plan

what comparisons do we need to perform to test Hypothesis 2?

# planning the comparisons

- your main effect comparisons should be derived from your ***a priori*** **hypotheses**

- some researchers argue that the comparisons should meet all three conditions for **orthogonality** (i.e., independence)

→***BUT your primary consideration should be your hypotheses – conduct the comparisons needed to test them fully!!!***

→***orthogonality is just a bonus if you can get it!***

# (1) analysis plan for main effect comparisons

- the researcher is interested in the following comparisons (*a priori*):
  - zero (i.e., no drug) vs. small, moderate & large (i.e., any drug)
  - small vs. moderate & large
- we will also add the following comparison to make an orthogonal set of 3 contrasts:
  - moderate vs. large

# setting up the contrasts

- have a go at completing the table below
- fill in the:
  - marginal means (top of table)
  - contrast weights for each of the 3 contrasts we are going to perform

| Drug Dosage: | Zero | Small | Moderate | Large |
|---|---|---|---|---|
| Means: | | | | |
| | | | | |
| Contrast 1 | | | | |
| Contrast 2 | | | | |
| Contrast 3 | | | | |

# analysis plan for linear contrasts

→ *answers*

These are the ***marginal means*** for drug dosage

| Drug Dosage: | Zero | Small | Moderate | Large |
|---|---|---|---|---|
| Means: | 8.00 | 12.30 | 10.00 | 9.90 |
| | | | | |
| Contrast 1 | **3** | **-1** | **-1** | **-1** |
| Contrast 2 | **0** | **2** | **-1** | **-1** |
| Contrast 3 | **0** | **0** | **1** | **-1** |

These are the ***contrast weights***
for each of the 3 contrasts we posed

# orthogonality condition #1: no more than *k* - 1 contrasts

The drug dosage factor had 4 levels (i.e., $k = 4$)

| Drug dosage: | Zero | Small | Moderate | Large |
|---|---|---|---|---|
| Mean: | 8.00 | 12.30 | 10.00 | 9.90 |

| | | | | |
|---|---|---|---|---|
| Contrast 1 | 3 | -1 | -1 | -1 |
| Contrast 2 | 0 | 2 | -1 | -1 |
| Contrast 3 | 0 | 0 | 1 | -1 |

***If*** we wanted to be able to show that our comparisons were orthogonal, we would not do more than 3 contrasts (i.e., $k - 1 = 4 - 1 = 3$)

# orthogonality condition #2: the sum of products of weights on 2 lines = 0

| | Zero | | Small | | Moderate | | Large |
|---|---|---|---|---|---|---|---|
| Drug dosage: | Zero | | Small | | Moderate | | Large |
| Mean: | 8.00 | | 12.30 | | 10.00 | | 9.90 |
| Contrast 1 | 3 | | -1 | | -1 | | -1 |
| | x | + | x | + | x | + | x |
| Contrast 2 | 0 | | 2 | | -1 | | -1 |
| Contrast 3 | 0 | | 0 | | 1 | | -1 |

(3 x 0 = **0**) + (-1 x 2 = **-2**) + (-1 x -1 = **1**) + (-1 x -1 = **1**) = 0

***If*** we wanted to show that our comparisons were orthogonal, we would do this for each possible pairing of the contrasts i.e., we'd also do this for (1) C1 & C3, and (2) C2 & C3

# orthogonality condition #3: weights within a contrast sum to 0

| Drug dosage: | Zero | Small | Moderate | Large |
|---|---|---|---|---|
| Mean: | 8.00 | 12.30 | 10.00 | 9.90 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Contrast 1 | 3 | + | -1 | + | -1 | + | -1 | = | 0 |
| Contrast 2 | 0 | + | 2 | + | -1 | + | -1 | = | 0 |
| Contrast 3 | 0 | + | 0 | + | 1 | + | -1 | = | 0 |

***→ This is always important to check. Any contrast that does <u>not</u> sum to zero is <u>not</u> a valid contrast.***

# the Bonferroni correction

**to test for significance we can either:**

**(a) test against a standard *t*-table**

**or**

**(b) test against a Bonferroni correction *t*-table (to adjust for familywise error rate)**

***whether you need to make the correction or not depends on three things:***

1. whether you decided to do the comparisons "*a priori*" or "*post hoc*"
2. how many comparisons you're doing
3. whether you want to be "conservative" or "liberal"

→ *more details on the next slide...*

# correction or no correction?

**the following is discussed in more detail in your workbooks (pp. 37-38)**

**Q1: Were my predictions for these comparisons made *post hoc* (i.e., after I performed the initial data analysis)?**

YES: Do a Bonferroni correction.

NO: Go to Q2.

----------------------------------------

**Q2: Am I performing 5 or more comparisons?**

YES: Do a Bonferroni correction.

NO: Go to Q3.

----------------------------------------

**Q3: Do I want to be conservative (rather than liberal)?**

YES: Do a Bonferroni correction.

NO: Don't need to do a Bonferroni correction (i.e., leave unadjusted).

# the critical *t* value

since:

(1) our predictions were made *a priori* (with the exception of the 3rd contrast which was included purely to achieve orthogonality),

(2) we're only planning to perform 3 comparisons (i.e., less than 5), and

(3) the dataset is not overly large, nor was the research novel or exploratory in nature (i.e., so there is no real need for us to be conservative)

→ **we will not be making use of any corrections**

we use the degrees of freedom (*df*) for the omnibus error term, so in our case:

$\boldsymbol{t_{crit_{\alpha = .05}}(32) = \pm 2.037}$ (as determined from *t*-tables)

# calculating linear contrasts

- have a go at doing the contrast calculations
  - use the two lots of formulae presented below (i.e., for *L* then *t*, for each contrast in turn)

$$L = \Sigma a_j \overline{X}_j$$

where $a_j$ = the contrast weight for a given group,
$\overline{X}$ = the mean for that same group, and
∑ = the sum of these products

$$t = \frac{L}{\sqrt{\frac{\Sigma a_j^2 \times MS_{error}}{n \times s}}}$$

where *L* = the value calculated above from the 1st equation,
*n* = the number of observations per cell, and
*s* = the number of levels of sex

# calculating linear contrasts

**Contrast 1** $L$ = (3 x 8.000) + (-1 x 12.300) + (-1 x 10.000) + (-1 x 9.900)

= 24.000 – 12.300 – 10.000 - 9.900 = **-8.200**

$$t = \frac{-8.200}{\sqrt{\frac{(9+1+1+1)\times(1.400)}{5\times 2}}} =$$ **-6.326**

**Contrast 2** $L$ = (0 x 8.000) + (2 x 12.300) + (-1 x 10.000) + (-1 x 9.900)
= **4.700**

$$t = \frac{4.700}{\sqrt{\frac{(4+1+1)\times(1.400)}{5\times 2}}} =$$ **5.128**

**Contrast 3** $L$ = (0 x 8.000) + (0 x 12.300) + (1 x 10.000) + (-1 x 9.900)
= **0.100**

$$t = \frac{0.100}{\sqrt{\frac{(1+1)\times(1.400)}{5\times 2}}} =$$ **0.189**

# results of linear contrasts

remember that the critical cut-off value is

$t_{crit_{\alpha = .05}}(32) = \pm 2.037$

***so...only contrasts 1 & 2 are significant***

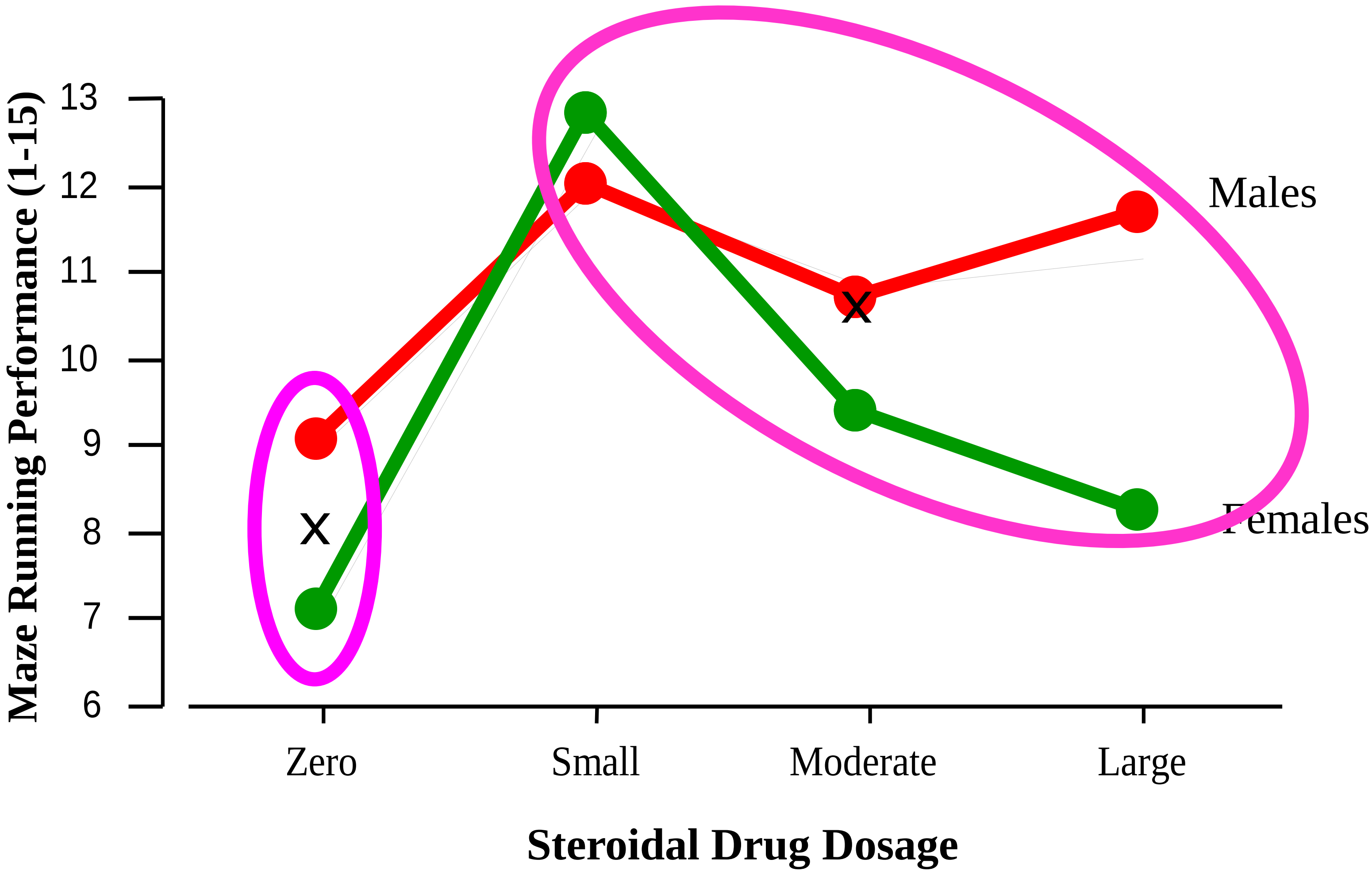
# interpreting the results of our linear contrasts

- what are we actually testing with these linear contrasts?

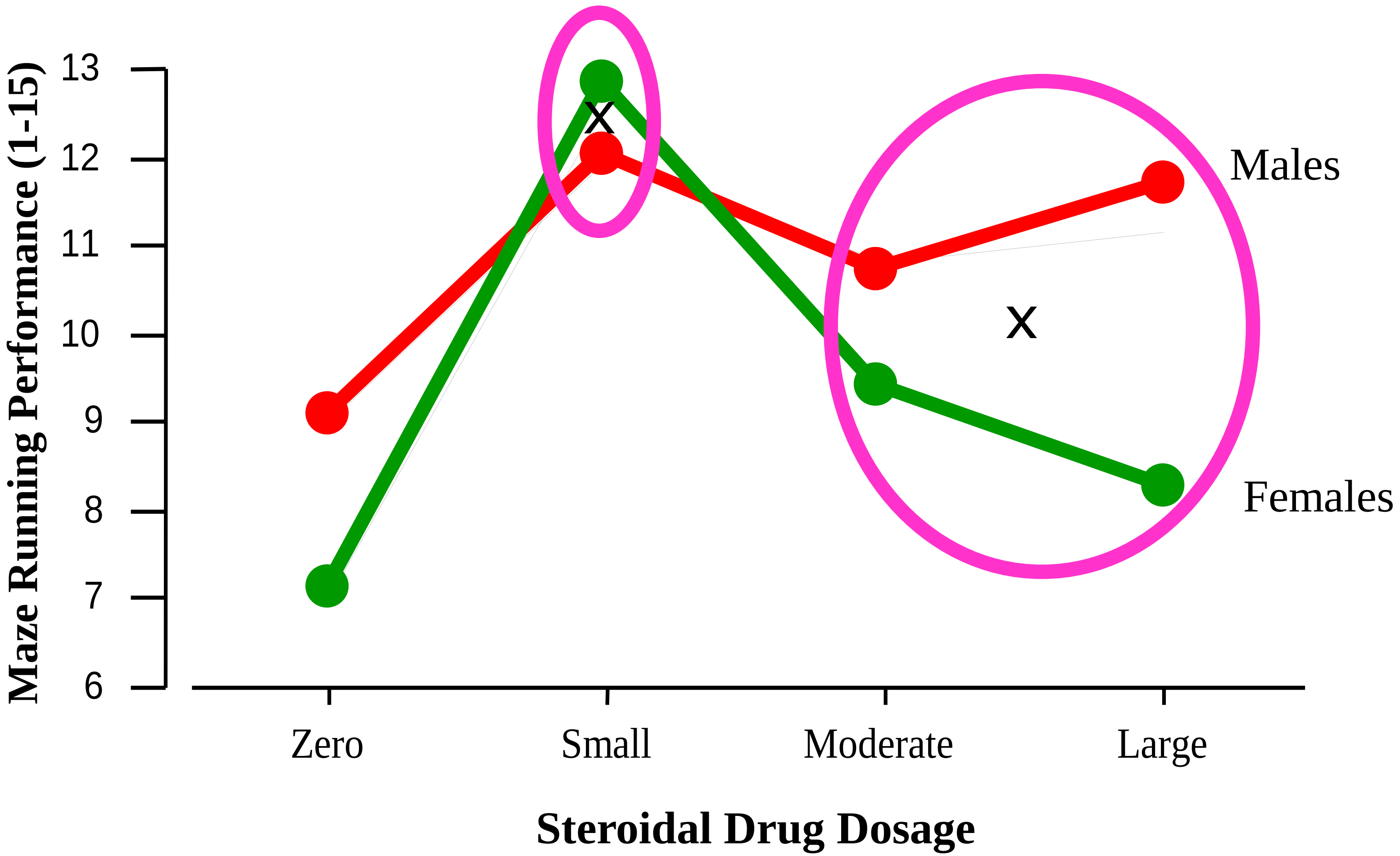
- what do the results actually tell us?

→ *well, let's have a look…*

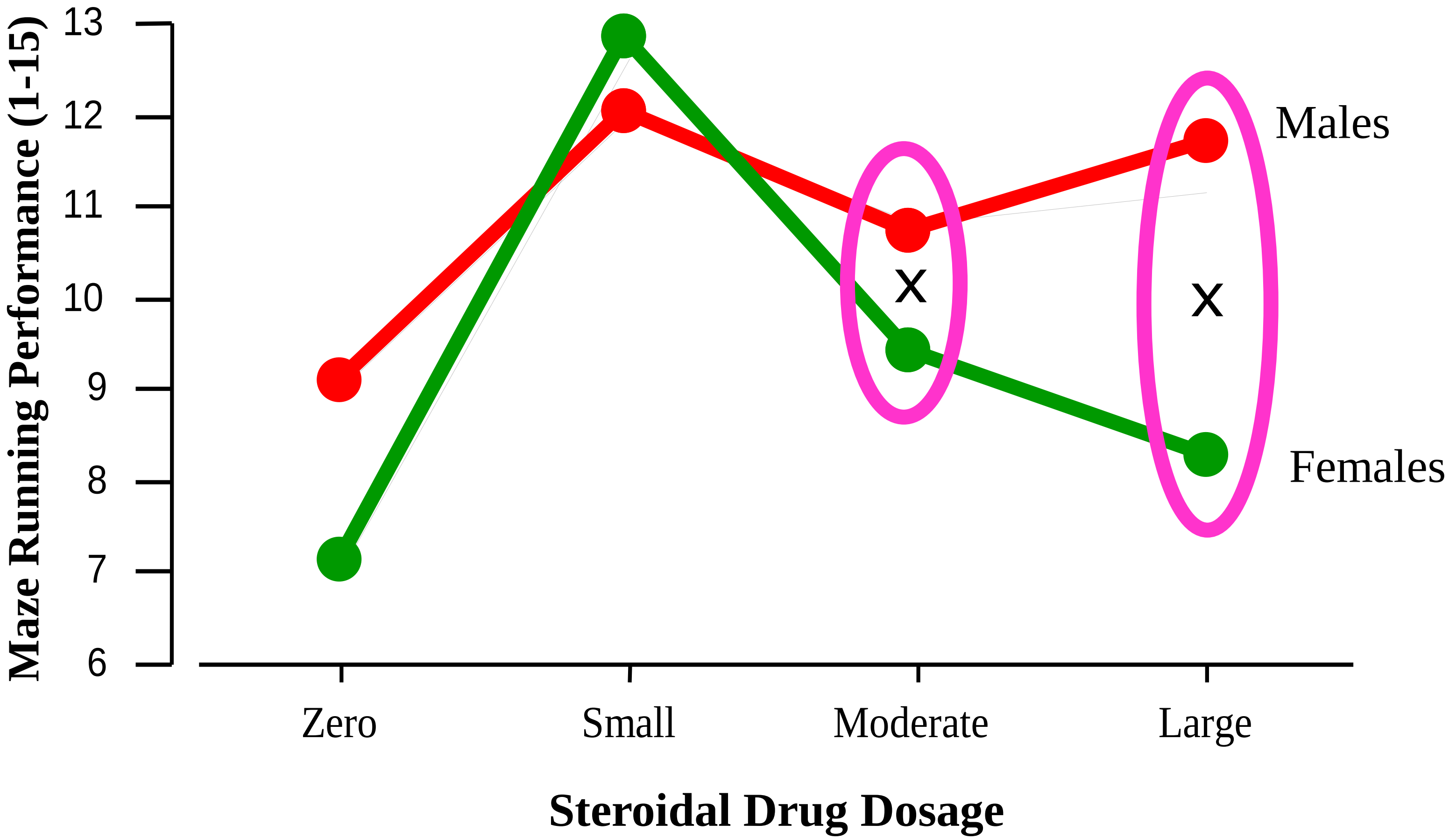
# comparison 1: zero compared to average of small, moderate & large



# comparison 2: small compared to average of moderate & large



# comparison 3: moderate compared to large



# writing up main effects with > 2 levels

1. **State what kind of ANOVA was performed, & what the factors/ IVs & DV were (inc. the levels of each factor/ IV)**
2. **Report results of the main effect**
   – State its significance
   – Be sure to give statistics (i.e., *F, df*, *p*, & effect size) to back this up
3. **If significant, report main effect comparisons**
   – What analyses did you use to run these main effect comparisons?
     - I.e., Did you use pairwise or linear comparisons? How many? Did you use a Bonferroni correction or no correction? What was the α-level employed?

   – What were the results?
     - Be sure to include sig./non-sig., direction of effect (as appropriate), means, *SDs*, *& p*-values

# the write up…

Results of a 2 (sex: male, female) x 4 (drug dosage: zero, small, moderate, large) between groups factorial ANOVA on maze running performance revealed a significant main effect of drug dosage, $F(3, 32) = 22.12$, $p < .001$, $\eta^2 = .51$. This was followed up with a series of three linear contrasts, each evaluated against $\alpha = .05$. The mean performance score for any drug dosage (small, moderate, or large; $M = 10.73$, $SD = 1.50$) was found to be significantly higher than that for the zero dosage ($M = 8.00$, $SD = 1.56$), $t(32) = 6.33$, $p < .05$. A significant difference in performance was also found between the small ($M = 12.30$, $SD = 0.95$) and a moderate or large dose ($M = 9.95$, $SD = 1.78$), such that rats receiving a small dosage performed better than those receiving moderate or large doses, $t(32) = 5.13$, $p < .05$. Maze running performance for the moderate ($M = 10.00$, $SD = 1.33$) and large doses ($M = 9.90$, $SD = 2.23$), however, did not differ significantly, $t(32) = 0.19$, *ns*.

# the write up…

Results of a 2 (sex: male, female) x 4 (drug dosage: zero, small, moderate, large) between groups factorial ANOVA on maze running performance revealed a significant main effect of drug dosage, *F*(3, 32) = 22.12, *p* < .001, $\eta^2$ = .51. This was

- Specify the analysis
- Detail the main effect results of the ANOVA

zero dosage (*M* = 8.00, *SD* = 1.56), *t*(32) = 6.33, *p* < .05. A significant difference in performance was also found between the small (*M* = 12.30, *SD* = 0.95) and a moderate or large dose (*M* = 9.95, *SD* = 1.78), such that rats receiving a small dosage performed better than those receiving moderate or large doses, *t*(32) = 5.13, *p* < .05. Maze running performance for the moderate (*M* = 10.00, *SD* = 1.33) and large doses (*M* = 9.90, *SD* = 2.23), however, did not differ significantly, *t*(32) = 0.19, *ns*.

# the write up…

Results of a 2 (sex: male, female) x 4 (drug dosage: zero, small, moderate, large) between groups factorial ANOVA on maze running performance revealed a significant main effect of drug dosage, $F(3, 32) = 22.12$, $p < .001$, $\eta^2 = .51$. This was followed up with a series of three linear contrasts, each evaluated against $\alpha = .05$. The mean performance score for

- If you followed up the omnibus test (because it was significant AND had > 2 levels), specify what you did

**NOTE: This write-up is for the hand-calculated <u>linear contrasts</u>, not the pairwise comparisons in SPSS!**

9.90, *SD* = 2.23), however, did not differ significantly, $t(32) = 0.19$, *ns*.

# the write up…

This is a pooled *M* & *SD*

Results of a 2 (sex: male, female) x 4 (drug dosage: zero, small, moderate, large) between groups factorial ANOVA on maze running performance revealed a significant main effect of drug dosage, $F(3, 32) = 22.12$, $p < .001$, $\eta^2 = .51$. This was followed up with a series of three linear contrasts, each evaluated against $\alpha = .05$. The mean performance score for any drug dosage (small, moderate, or large; $M = 10.73$, $SD = 1.50$) was found to be significantly higher than that for the zero dosage ($M = 8.00$, $SD = 1.56$), $t(32) = 6.33$, $p < .05$. A

- Specify IV conditions and DV means & *SD*s
- Provide info on the **DIRECTION OF EFFECT**
- Include *t*(df) and *p* value for each comparison
- NOTE: *t*-values are not actually necessary

→**NOTE: Exact *p* values are not reported here since these are results from the hand-calculated linear contrasts. If you had used pairwise comparisons from SPSS, you should report exact *p*-values (again, *t* is optional).**

# the write up…

Results of a 2 (sex: male, female) x 4 (drug dosage: zero, small, moderate, large) between groups factorial ANOVA on maze running performance revealed a significant main effect of drug dosage, $F(3, 32) = 22.12$, $p < .001$, $\eta^2 = .51$. This was followed up with a series of three linear contrasts, each evaluated against $\alpha = .05$. The mean performance score for any drug dosage (small, moderate, or large; $M = 10.73$, $SD =$

- This is how you report a hand-calculated pairwise comparison – it's the same principle as we saw before, but now you are only comparing 2 means (again, *t* is optional now)

doses, $t(32) = 5.13$, $p < .05$. Maze running performance for the moderate ($M = 10.00$, $SD = 1.33$) and large doses ($M = 9.90$, $SD = 2.23$), however, did not differ significantly, $t(32) = 0.19$, *ns*.

**NOTE:** *M*s and *SD*s are reported only once in the write-up!

# (2) following up interactions

- Simple effects are used to follow up significant interactions
  - The factorial interaction can only tell you there is a significant difference “somewhere” among the cell means
- A simple effect compares cell means of an IV ***at each level*** of another IV
  - Question being addressed: “Is there an effect of an IV at each level of the other IV?”
- Let’s briefly see how simple effects differ from main effects…

# simple effects vs. main effects

- In the presence of an interaction, simple effects provide more information than main effects
- Main effects tell you about mean differences in the levels of an IV ***averaged over*** the levels of other IV(s)
- In contrast, simple effects tell you about mean differences in the levels of an IV ***at each*** level of other IV(s)
- So with reference to our data…

The **s**imple effects of sex compare the **c**ell means of **sex**, ***at each level*** of drug dosage

| | | **Drug Dosage** | | | | |
|---|---|---|---|---|---|---|
| **Sex** | Zero | Small | Moderate | Large | $T_{S_i}$ | $\bar{X}_{S_i}$ |
| Male | 10 | 10 | 12 | 10 | | |
| | 10 | 13 | 10 | 11 | | |
| | $T_{11} = 45$ | $T_{12} = 60$ | $T_{13} = 54$ | $T_{14} = 58$ | | |
| | $\bar{X}_{11} = 9.00$ | $\bar{X}_{12} = 12.00$ | $\bar{X}_{13} = 10.80$ | $\bar{X}_{14} = 11.60$ | 217 | 10.85 |
| Female | 6 | 12 | 9 | 9 | | |
| | 7 | 13 | 9 | 10 | | |
| | 6 | 13 | 10 | 6 | | |
| | 8 | 12 | 8 | 9 | | |
| | 8 | 13 | 10 | 7 | | |
| | $T_{21} = 35$ | $T_{22} = 63$ | $T_{23} = 46$ | $T_{24} = 41$ | | |
| | $\bar{X}_{21} = 7.00$ | $\bar{X}_{22} = 12.60$ | $\bar{X}_{23} = 9.20$ | $\bar{X}_{24} = 8.20$ | 185 | 9.25 |
| $T_{D_j}$ | 80 | 123 | 100 | 99 | 402 | |
| $\bar{X}_{D_j}$ | 8.00 | 12.30 | 10.00 | 9.90 | | 10.05 |

**Drug Dosage**

| **Sex** | Zero | Small | Moderate | Large | $T_{S_i}$ | $\bar{X}_{S_i}$ |
|---|---|---|---|---|---|---|
| Male | 10 | 10 | 12 | 10 | | |
| | $T_{11} = 45$ | $T_{12} = 60$ | $T_{13} = 54$ | $T_{14} = 58$ | | |
| | $\bar{X}_{11} = 9.00$ | $\bar{X}_{12} = 12.00$ | $\bar{X}_{13} = 10.80$ | $\bar{X}_{14} = 11.60$ | 217 | 10.85 |
| Female | 6 | 12 | 9 | 9 | | |
| | 7 | 13 | 9 | 10 | | |
| | 6 | 13 | 10 | 6 | | |
| | 8 | 12 | 8 | 9 | | |
| | 8 | 13 | 10 | 7 | | |
| | $T_{21} = 35$ | $T_{22} = 63$ | $T_{23} = 46$ | $T_{24} = 41$ | | |
| | $\bar{X}_{21} = 7.00$ | $\bar{X}_{22} = 12.60$ | $\bar{X}_{23} = 9.20$ | $\bar{X}_{24} = 8.20$ | 185 | 9.25 |
| $T_{D_j}$ | 80 | 123 | 100 | 99 | 402 | |
| $\bar{X}_{D_j}$ | 8.00 | 12.30 | 10.00 | 9.90 | | 10.05 |

The **s**imple effects of dosage compare the **c**ell means of **drug dosage**, ***at each level*** of sex

# interaction follow up tests: simple effects

Imagine that our **Hypothesis 3** read:

The effect of drug dosage will differ for males compared to females. While both sexes will exhibit increased performance when they receive the drug than when they do not, the particular benefits of the small drug dosage (compared to moderate and large doses) will be more noticeable in female rats than in male rats.

# from hypotheses to analyses: developing the analysis plan

Based on hypothesis 3, we expect a significant interaction (which was shown, as seen in the workbook ANOVA summary table on pp. 49). This will need to be followed up....

***To test hypothesis 3, which set of simple effects do we need to conduct?***

# from hypotheses to analyses: developing the analysis plan

The answer is:
the simple effects of
DRUG DOSAGE

# simple effects results

- Results for the simple effects of drug dosage were found to be significant both for males and for females, as seen in the table below:

**Univariate Tests**

Dependent Variable: Performance

| Sex | | Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|---|---|---|---|---|---|---|---|
| Male | Contrast | 26.550 | 3 | 8.850 | 6.321 | .002 | .372 |
| | Error | 44.800 | 32 | 1.400 | | | |
| Female | Contrast | 86.950 | 3 | 28.983 | 20.702 | .000 | .660 |
| | Error | 44.800 | 32 | 1.400 | | | |

Each F tests the simple effects of Dosage within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

# but we still don’t have the full story…

- Simple effects don’t always tell you exactly where the cell mean differences are…
- Therefore, **simple comparisons** are needed to follow up significant simple effects on variables with > 2 levels
  - This is the case here, as drug dosage has 4 levels

# (3) simple comparisons

- Are exactly like main effect comparisons, ***BUT*** they follow up the effects of a factor ***within each level*** of the other factor(s)

- Can use linear contrasts or pairwise comparisons, just as when following up main effects

- Your simple comparisons may be determined *a priori*, & – if possible – they should meet the conditions for *orthogonality*
  (for a recap of orthogonality, look back to the earlier slides)

# from hypotheses to analyses: developing the analysis plan

According to hypothesis 3, which linear comparisons do we need to perform in order to follow up the simple effects?

As a reminder, H3 states:

The effect of drug dosage will differ for males compared to females. While both sexes will exhibit increased performance when they receive the drug than when they do not, the particular benefits of the small drug dosage (compared to moderate and large doses) will be more noticeable in female rats than in male rats.

# analysis plan

- We need the following comparisons (*a priori*) for each sex group (i.e., for males & females, separately) to address H3 fully:

  (1) Zero (i.e., no drug) vs. small, moderate & large (i.e., some drug)

  (2) Small vs. moderate & large

- Once again, to achieve orthogonality (because it is possible), we will also throw in the following:

  (3) Moderate vs. large

This will give us the *full* set of orthogonal contrasts ($k$ - 1)

# linear contrasts: males

The contrast weights for males would look like:

| | | | | |
|---|---|---|---|---|
| Drug Dosage: | Zero | Small | Moderate | Large |
| Means: | 9.00 | 12.00 | 10.80 | 11.60 |
| Contrast 1 | **3** | **-1** | **-1** | **-1** |
| Contrast 2 | **0** | **2** | **-1** | **-1** |
| Contrast 3 | **0** | **0** | **1** | **-1** |

These are the **cell means** of drug dosage for **males**

# calculations for linear contrasts: males

$$L = \sum a_j \overline{X}_j$$

$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 * MS_{error}}{n}}}$$

**NOTE: The equation for simple effects is slightly different to main effects because it is based on a different number of observations**

**Contrast 1** $L$ = 27.000 – 12.000 - 10.800 - 11.600

= **-7.400**

$$t = \frac{-7.400}{\sqrt{\frac{(9+1+1+1)*(1.400)}{5}}} = \textbf{-4.037}$$

**Contrast 2** $L$ = 24.000 - 10.800 - 11.600

= **1.600**

$$t = \frac{1.600}{\sqrt{\frac{(4+1+1)*(1.400)}{5}}} = \textbf{1.234}$$

**Contrast 3** $L$ = 10.800 - 11.600

= **-0.800**

$$t = \frac{-0.800}{\sqrt{\frac{(1+1)*(1.400)}{5}}} = \textbf{-1.069}$$

# results for linear contrasts: males

- Again, since there are only 3 comparisons & these were predicted *a priori*, we're not going to perform any adjustments/ corrections

- For the *df*, use *df* for error

- $t_{\text{crit}\ \alpha = .05}(32) = \pm\ 2.037$

***So...only contrast 1 is significant regarding the effect of drug dosage for males***

# linear contrasts - females

The contrast weights for females would look like:

| Drug Dosage: | Zero | Small | Moderate | Large |
|---|---|---|---|---|
| Means: | 7.00 | 12.60 | 9.20 | 8.20 |
| Contrast 1 | **3** | **-1** | **-1** | **-1** |
| Contrast 2 | **0** | **2** | **-1** | **-1** |
| Contrast 3 | **0** | **0** | **1** | **-1** |

These are the **cell means** of drug dosage for **females**

# calculations for linear contrasts: females

→ ***The formulae for L and t are the same as that for males***

**Contrast 1** $L$ = 21.000 – 12.600 – 9.200 – 8.200

= **-9.000**

$$t = \frac{-9.000}{\sqrt{\frac{(9+1+1+1)*(1.400)}{5}}} = \textbf{-4.910}$$

**Contrast 2** $L$ = 25.200 – 9.200 – 8.200

= **7.800**

$$t = \frac{7.800}{\sqrt{\frac{(4+1+1)*(1.400)}{5}}} = \textbf{6.018}$$

**Contrast 3** $L$ = 9.200 – 8.200

= **1.000**

$$t = \frac{1.000}{\sqrt{\frac{(1+1)*(1.400)}{5}}} = \textbf{1.336}$$

# results for linear contrasts: females

- Again, since there are only 3 comparisons & these were predicted *a priori*, we're not going to perform any adjustments/ corrections

- For the *df*, use *df* for error

- $t_{\text{crit } \alpha = .05}\,(32) = \pm\, 2.037$

***So...contrasts 1 & 2 are significant regarding the effect of drug dosage for females***

# writing up the interaction: omnibus test

In addition, a significant Sex x Drug Dosage interaction on maze running performance was revealed, $F(3, 32) = 4.91$, $p = .006$, $\eta^2 = .11$.

# writing up the interaction: 1st simple effect → simple comparisons

This was followed up by performing the simple effects of drug dosage.  The simple effect of dosage was significant for males, $F(3, 32) = 6.32$, $p = .002$, $\eta^2 = .14$.  This was, in turn, followed up with a set of three linear contrasts, each evaluated against $\alpha = .05$.

**NOTE: This write-up is for the hand-calculated linear contrasts.  If you had conducted pairwise comparisons in SPSS, you would report that pairwise comparisons were performed, how many, if Bonferroni corrections were used and the α-level employed.**

# writing up the interaction: simple comparisons for males

Pooled *M* & *SD* values

For male rats, maze running performance for any drug dosage (small, moderate, or large, *M* = 11.47, *SD* = 1.22) was found to be significantly higher than that for the zero dosage (*M* = 9.00, *SD* = 1.41), $t(32) = 4.04$, $p < .05$. No significant difference in performance was found between a small (*M* = 12.00, *SD* = 1.22) and a moderate or large dose (*M* = 11.20, *SD* = 1.22), $t(32) = 1.24$, *ns*. Likewise, the performance of rats who received moderate (*M* = 10.80, *SD* = 1.30) and large drug dosages (*M* = 11.60, *SD* = 1.14) did not differ significantly, $t(32) = -1.07$, *ns*.

**NOTE: This write-up is for the hand-calculated linear contrasts. If you had conducted pairwise comparisons in SPSS, you would report the exact *p* values even for *ns* results. Again, the *t*-values are optional reporting these days.**

# writing up the interaction: simple comparisons for females

A simple effect of drug dosage also emerged for females rats, $F(3,32) = 20.70$, $p < .001$, $\eta^2 = .47$…

- Try writing the rest of this yourself using the previous slides as a guide, now that you understand what write-up components are required! ☺